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Hands-on Exercise #2: Run-time Analysis

For each of the following program fragments,

1. count the total number of operations. What is the big O value?
2. What is the return value if A = {1, 2, 3} and B = {1, 3, 5}?
   1. **Algorithm Ex1(A):**

Input: An array A storing n ≥ 1 integers. Output: The sum of the elements in A.

s ← A[0]

**for** i ← 1 to n - 1 **do**

s ← s + A[i]

**return** s

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | # of operations | Current total | Simplified total | Big O |
| s ← A[0] | 1 | 1 | 1 |  |
| for i ← 1 to n - 1 do | (n-1) | 1+(n-1) | n |  |
| s ← s + A[i] | (n-1) | 1+(n-1) + (n-1) | n + (n-1) |  |
| return s | 1 | 1+(n-1) + (n-1) + 1 | 2n | n |

|  |  |  |  |
| --- | --- | --- | --- |
| A = {1, 2, 3} | s | i | n |
| 1, 2, 3 | 1 | 1 | 3 |
|  | 1+2=3 | 2 |  |
|  | 3+3=6 |  |  |

* 1. **Algorithm Ex2(A):**

Input: An array A storing n ≥ 1 integers.

Output: The sum of the elements at even cells in A. s ← A[0]

**for** i ← 2 to n - 1 by increments of 2 **do**

s ← s + A[i]

**return** s

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | # of operations | Current total | Simplified total | Big O |
| s ← A[0] | 1 | 1 | 1 |  |
| for i ← 2 to n - 1 by increments of 2 do | (n-1) | 1+(n-1) | n |  |
| s ← s + A[i] | (n-1) | 1+(n-1) + (n-1) | n + (n-1) |  |
| return s | 1 | 1+(n-1) + (n-1) + 1 | 2n | n |

|  |  |  |  |
| --- | --- | --- | --- |
| A = {1, 2, 3} | s | i | n |
| 1, 2, 3 | 1 | 2 | 3 |
|  | 1+3=4 | 2 |  |
|  |  |  |  |

* 1. **Algorithm Ex3(A):**

Input: An array A storing n ≥ 1 integers. Output: The sum of the prefix sums in A. s ← 0

**for** i ← 0 to n – 1 **do**

s ← s + A[0]

**for** j ← 1 to i **do**

s ← s + A[j]

**return** s

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A = {1, 2, 3} | s | i | j | n |
| 1, 2, 3 | 0 | 0 | ? | 3 |
| s ← s + A[0] | 0+1=1 | 0 | ? |  |
| s ← s + A[j] | 1+2=3 | 0 | 1 |  |
| s ← s + A[0] | 3+1 = 4 | 1 | 1 |  |
| s ← s + A[j] | 4+1 = 5 | 1 | 1 |  |
| s ← s + A[0] | 5+1=6 | 2 | 1 |  |
| s ← s + A[j] | 6+1=7 | 2 | 1 |  |
| s ← s + A[j] | 7+2=9 | 2 | 2 |  |
| s ← s + A[0] | 9+1=10 | 3 | 2 |  |
| s ← s + A[j] | 10+1=11 | 3 | 1 |  |
| s ← s + A[j] | 11+2=13 | 3 | 2 |  |
| s ← s + A[j] | 11+3=16 | 3 | 3 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | # of operations | Current total | Simplified total | Big O |
| s ← 0 | 1 | 1 | 1 |  |
| for i ← 0 to n – 1 do | (n-1) | 1+(n-1) | n |  |
| s ← s + A[0] | (n-1) | 1+(n-1) + (n-1) | n + (n-1) |  |
| for j ← 1 to i do | (n-1) | [1+(n-1) + (n-1)] \* \*[(n-1)] |  |  |
| s ← s + A[j] |  | [1+(n-1) + (n-1)] \* \*[(n-1) +(n-1)] |  |  |
| return s | 1 | [1+(n-1) + (n-1)] \* \*[(n-1) +(n-1)] + 1 | n \* n | N^2 |

* 1. **Algorithm Ex4(A):**

Input: An array A storing n ≥ 1 integers. Output: The sum of the prefix sums in A. s ← A[0]

t ← s

**for** i ← 1 to n - 1 **do**

s ← s + A[i] t ← t + s

**return** t

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A = {1, 2, 3} | s | i | t | n |
| s ← A[0]  t ← s | 1 | 0 | 1 | 3 |
| s ← s + A[i] | 1+2=3 | 1 | 1 |  |
| t ← t + s | 3 | 1 | 1+3=4 |  |
| s ← s + A[i] | 3+3=6 | 2 | 4 |  |
| t ← t + s | 6 | 2 | 4+6=10 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | # of operations | Current total | Simplified total | Big O |
| s ← A[0] | 1 | 1 | 1 |  |
| t ← s | 1 | 1+1+(n-1) | n |  |
| for i ← 1 to n - 1 do | (n-1) | 1+1+(n-1) | n + (n-1) |  |
| s ← s + A[i] | (n-1) | 1+1+(n-1) + (n-1) |  |  |
| t ← t + s | (n-1) | 1+1+(n-1) + (n-1) +(n-1) |  |  |
| return t | 1 | 1+1+(n-1) + (n-1) +(n-1) +1 | n | O(n) |

* 1. **Algorithm Ex5(A):**

Input: Arrays A and B each storing n ≥ 1 integers.

Output: The number of elements in B equal to the sum of the prefix sums in A.

c ← 0

**for** i ← 0 to n - 1 **do**

s ← 0

**for** j ← 0 to i **do**

s ← s + A[0]

**for** k ← 1 to j **do**

s ← s + A[k]

**if** B[i] = s **then**

c ← c + 1

**return c**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A={1,2,3}** | **B={1,3,5}** | **c** | **i** | **j** | **k** | **s** |
| **c ← 0**  **for i ← 0 to n - 1 do**  **s ← 0** |  | **0** | **0** | **?** | **?** | **0** |
| **for j ← 0 to i do** | **0** | **0** | **0** | **0** | **?** | **0** |
| **s ← s + A[0]** |  | **0** | **0** | **0** | **?** | **0+1=1** |
| **for k ← 1 to j do** |  | **0** | **0** | **0** | **1** |  |
| **s ← s + A[k]** |  |  |  |  |  |  |
| **if B[i] = s then** | **B[0]=1** |  |  |  |  |  |
| **c ← c + 1** |  |  |  |  |  |  |
| **for i ← 0 to n - 1 do** | **0** | **0** | **1** | **0** | **?** | **0** |
| **s ← 0** |  |  | **1** | **0** |  | **0** |
| **for j ← 0 to i do** |  |  | **1** | **0** |  | **0** |
| **s ← s + A[0]** |  |  | **1** | **0** |  | **0+1=1** |
| **Skip to** |  |  |  |  |  |  |
| **s ← s + A[0]** |  |  | **1** | **1** |  | **1+1=2** |
| **for k ← 1 to j do** |  |  | **1** | **1** | **1** |  |
| **s ← s + A[k]** |  |  | **1** | **1** | **1** | **2+2=4** |
| **if B[i] = s then** | **B[1]=3** |  |  |  |  |  |
| **c ← c + 1** |  |  |  |  |  |  |
| **for i ← 0 to n - 1 do** |  |  | **2** | **1** | **1** |  |
| **s ← 0** |  |  | **2** | **1** | **1** | **0** |
| **for j ← 0 to i do** |  |  | **2** | **0** | **1** |  |
| **s ← s + A[0]** |  |  |  |  |  |  |
| **Skip to** |  |  |  |  |  |  |
| **s ← s + A[0]** |  |  | **2** | **1** | **1** |  |
| **for k ← 1 to j do** |  |  | **2** | **1** | **1** |  |
| **s ← s + A[k]** |  |  | **2** | **1** | **1** | **0+3=3** |
| **if B[i] = s then** | **B[3]=n/a** |  |  |  |  |  |
| **c ← c + 1** |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| **for j ← 0 to i do** |  |  | **2** | **2** | **1** |  |
| **s ← s + A[0]** |  |  | **2** | **2** | **1** | **3+1=4** |
| **for k ← 1 to j do** |  |  | **2** | **2** | **1** |  |
| **s ← s + A[k]** |  |  | **2** | **2** | **1** | **4+2=6** |
| **if B[i] = s then** | **B[2]=5** |  | **2** | **2** | **1** | **6** |
| **c ← c + 1** |  |  |  |  |  |  |
| **for k ← 1 to j do** |  |  | **2** | **2** | **2** |  |
| **s ← s + A[k]** |  |  | **2** | **2** | **2** | **6+3=9** |
| **if B[i] = s then** |  |  |  |  |  |  |
| **c ← c + 1** |  |  |  |  |  |  |
|  |  | **0** |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | # of operations | Current total | Simplified total | Big O |
| c ← 0 | 1 | 1 | 1 |  |
| for i ← 0 to n - 1 do | (n-1) | 1+(n-1) | n |  |
| s ← 0 | (n-1) | 1+(n-1) +(n-1) | n + (n-1) |  |
| for j ← 0 to i do | (n-1) | 1+(n-1) + (n-1) +(n-1) \*(n-1) |  |  |
| s ← s + A[0] | (n-1) | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) |  |  |
| for k ← 1 to j do | 1 | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) \*(n-1) | n | O(n) |
| s ← s + A[k] |  | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) \*(n-1) +(n-1) |  |  |
| if B[i] = s then | 1 | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) \*(n-1) +(n-1) +1 |  |  |
| c ← c + 1 | 1 | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) \*(n-1) +(n-1) +1 +1 |  |  |
| return c | 1 | 1+(n-1) + (n-1) +(n-1) \*(n-1) +(n-1) \*(n-1) +(n-1) +1 +1 +1 | n\*n\*n | O(n^3) |